Variation of Parameters

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Ordinary Differential Equations - MA 266
Variation of Parameters

We introduce a method for solving non-homogeneous linear second order DE with constant coefficients

\[ a_2 y'' + a_1 y' + a_0 y = g(x) \]

where \( a_2, a_1, a_0 \) are constants. To solve we follow the next steps

1. We find the solution \( y_c \) of the complementary homogeneous

\[ a_2 y'' + a_1 y' + a_0 y = 0 \]

\[ y_c = c_1 y_1 + c_2 y_2 \]

2. We divide the DE by \( a_2 \) put it in standard form

\[ y'' + Py' + Qy = f(x) \]

3. We find functions \( u_1, u_2 \) given by

\[ u_1' = -\frac{y_2 f(x)}{W}, \quad u_2' = \frac{y_1 f(x)}{W} \]

where \( W \) is the Wronskian of \( y_1 \) and \( y_2 \).
Variation of Parameters

1. We find the solution $y_c$ of the complementary homogeneous
   $a_2 y'' + a_1 y' + a_0 y = 0$

   $y_c = c_1 y_1 + c_2 y_2$

2. We divide the DE by $a_2$ put it in standard form
   $y'' + Py' + Qy = f(x)$ to determine $f(x)$

3. We find functions $u_1, u_2$ given by

   $u_1' = -\frac{y_2 f(x)}{W}$, $u_2' = \frac{y_1 f(x)}{W}$

   where $W$ is the Wronskian of $y_1$ and $y_2$.

4. The general solution of the equation is

   $y = y_c + y_p$

   where $y_p$ is

   $y_p = u_1 y_1 + u_2 y_2$
Example

Solve

\[ y'' - 4y' + 4y = (x + 1)e^{2x} \]
Example

Solve

\[ 4y'' + 36y = \csc 3x \]
Example

Solve

\[ y'' + y = \sec x \]
Example

Solve

\[ y'' - 9y = \frac{9x}{e^{3x}} \]
Exercises

Exercises. Solve Exercises 1-22 from Page 162 of the Textbook