Initial-Value Problems

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Ordinary Differential Equations - MA 266
Initial-Value Problems

**Definition** The problem of solving an $n$th-order ODE subject to $n$ conditions specified at $x_0$ is

- Solve: $\frac{d^n y}{dx^n} = f(x, y, y', \ldots, y^{(n-1)})$
- Subject to: $y(x_0) = y_0, y'(x_0) = y_1, \ldots, y^{(n-1)}(x_0) = y_{n-1}$

where $y_0, y_1, \ldots y_{n-1}$ are arbitrary real constants, is called an $n$th-order initial-value problem (IVP).

The above conditions are called **initial conditions (IC)**.

Solving an $n$th-order initial-value problem frequently entails first finding an $n$-parameter family of solutions and then using the initial-conditions to determine the $n$ constants in this family.

The resulting solution is a **particular solution**.
Initial-Value Problem first order

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- Solve: $\frac{dy}{dx} = f(x, y)$
- Subject to: $y(x_0) = y_0$

Example.
a) Prove that $y = ce^x$ is a one-parameter family of solutions of the first-order equation $y' = y$.
b) Find a solution of the IVP consisting of this differential equation and the initial condition $y(0) = 3$.
c) Find a solution of the IVP consisting of this differential equation and the initial condition $y(1) = -2$.
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Initial-Value Problem second order

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- Solve: $\frac{d^2y}{dx^2} = f(x, y, y')$
- Subject to: $y(x_0) = y_0, y'(x_0) = y_1$

Example.

a) Prove that $y = c_1 \cos 4t + c_2 \sin 4t$ is a two-parameter family of solutions of the second-order equation $y'' + 16y = 0$.

b) Find a solution of the IVP consisting of this differential equation and the initial conditions

$$y\left(\frac{\pi}{2}\right) = -2, \quad y'\left(\frac{\pi}{2}\right) = 1$$
Initial-Value Problem second order

Example.

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\[
y\left(\frac{\pi}{2}\right) = -2, \quad y'(\frac{\pi}{2}) = 1
\]
Example

Given that \( y = \frac{1}{1 + c_1 e^{-x}} \) is a one-parameter family of solutions of the first-order DE \( y' = y - y^2 \). Find a solution of the first-order IVP consisting of this differential equation and the initial condition

a) \( y(0) = -\frac{1}{3} \)
b) \( y(-1) = 2 \)
Example

Given that $y = c_1 e^x + c_2 e^{-x}$ is a two-parameter family of solutions of the second-order DE $y'' - y = 0$. Find a solution of the second-order IVP consisting of this differential equation and the initial conditions.

$$y(0) = 1 \text{ and } y'(0) = 2$$
Exercises.

Exercises. Solve Exercises 3-10, 12-16 from Page 17 of the Textbook.